## Review of Ratio-Dependent Predator Prey System

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1. Build up the model

They transform a ratio-dependent predator-prey system (1.1) into a Gause-type predator-prey system (1.5).

Rescale system (1.1), let , then

And insert equation (1.1) into equation (1.2), we can get

Let , so are all larger or equal to 0. Let , then we can get the system of equations:

Finally, in the last step, they let (variable change), so that

Note that

Then we can get the main system of equations that the article uses:

1. Condition classification

The article finds conclusions and answers open questions of previous works about system (1.5) in terms of different conditions and talks about asymptotic behavior of the solutions which are not mentioned in previous works. There are 4 main conditions:

It’s worth noting that .

For (1), is equivalent to because ; And when , we have

For (2), it’s getting complicated so we list out the conditions showed in the article:

|  |  |
| --- | --- |
| Conditions (with ) | Conditions (with ) |
|  |  |
|  | , |
|  |  |
|  |  |
|  |  |
|  |  |

Form 1.

For (4), is covered in Form 1.

1. Summary of Purpose

The goal of this paper is to give an almost complete classification for the asymptotic behavior of the solutions of the model described above. It does so by organizing and explaining the previous work done on this model, as well as addressing the previously open questions related to it. These open questions are the following:

1. Is (1, 0) locally stable for condition (4) in the paper?
2. Is globally stable for condition (7) in the paper?
3. Is it true that if , then is globally stable?

We will refer to these as Open Questions 1, 2, and 3 respectively.

Theorem 2.1 answers the first open question of the model, and it states If and , then the equilibrium (1,0) is globally asymptotically stable for our system in . The proof of this theorem involves looking at the eigenvalues of the variational matrix, the use of the previous lemmas of this section, and observing separate cases for directional field charts.

Theorem 2.3 addresses our second open question, stating that , the positive equilibrium point, exists and is indeed globally asymptotically stable in for our model given the boundary conditions. The proof uses the use of the Lyapunov function

The final open question is answered by theorem 2.5. This theorem is proved using the logic of a former remark made about the boundary conditions, as well as some simple phase plane analysis.

Thus through these theorems, the paper accomplishes its goal of correctly answering the open questions surrounding this model. In the rest of the paper, they simply discuss more about the model, explaining the details of why this ratio-dependent model accomplishes what most non ratio-dependent models cannot show us. This model shows more interesting dynamics, and helps to give simple and plausible explanations of them as well.

1. Discussion of Other Theorems and Lemmas

In the article, there are some important theorems and lemmas which we haven’t discussed yet. We pick up three of them we are interest in. Lemma 2.2 gives an overall conclusion of the asymptotic behavior of and . The proof of Lemma 2.2 gives out the method of variation matrix of the system at the three points: and .

Theorem 2.8 and Theorem 2.9 describes the behavior of , the stable manifold of . These theorems use the condition labelled above in Form 1 as (\*\*). When , and , the manifold separates the space into two parts. Trajectories in the two areas will finally approach (0, 0) and , respectively. When and , and will be connected by , and the other trajectories will all approach (0, 0).

1. Actual meaning for the solutions

For the behaviors of all the solutions, we can classify them into three types:

1. One of the predator and prey will finally extinct
2. The predator and prey will extinct simultaneously
3. The predator and prey will coexist in the form of equilibrium points/ limit cycle.

For example, when the predator death rate is larger or equal to its maximal growth rate (), the predator will go to extinct; when predator death rate is less than its maximal growth rate (), and with the condition and , there are two possible development of the two species: if is small, they will coexist in the form of equilibrium.